UNIVERSITY OF TORONTO MISSISSAUGA
April 2020 FINAL EXAMINATION
CSC498H5S
Computational Complexity and Computability
Vincent Maccio
Duration - 24 hours

*This is a take home exam. You have 24 hours to complete this exam. You must submit a .pdf to MarkUs by 8:00pm April 20th. You do not need to copy the questions to your .pdf, only the answers are required. Make it very clear which questions you are answering. You have 24 hours, your submitted work should be extremely clear, easy to follow, and free of spelling and grammar errors. Do **not** submit photos of hand written answers.*

There are 5 questions on this exam. Each question is worth 10 marks. For each of the 5 questions you may write "I do not know." and receive 3/10 for that question. If you choose to do this, you must do it for the entire question. That is, you cannot write "I do not know." for a sub-question and receive 30% on that sub-question (the math is just too annoying).

Good Luck!

Question 1. [10 MARKS]

For each statement below, write True or False. There is no need to justify your answer.

1. If $\int f(x)dx = 1$, then $f(x)$ is a probability density function.
2. The exponential distribution is an IFR distribution
3. The maximum of two exponentially distributed random variables is exponentially distributed
4. The device with the highest expected service time will be the device with the highest utilization which will be the bottle-neck
5. If $\rho = 1$ the system is unstable
6. Using Mean Value Analysis you can determine if the number of jobs at a given node equals 0
7. If X is exponentially distributed, then $P(X > x) = P(X > x + t \mid X > x)$
8. The rate out of all states in an M/M/2 queue is $(\lambda + 2\mu)$
9. The original PageRank algorithm used an underlying continuous time Markov chain
10. If X is Pareto distributed, and $P(X > x) = p$, then $P(X > x + t \mid X > t > 0) > p$

Question 2. [10 marks]

Consider a single server system where jobs arrive following a Poisson process. Job service times are generally distributed. Determine a metric for this system you would use to measure the "fairness". Justify why you chose this metric. What do you think it means for the system to be fair? Identify and discuss a service policy which under your metric would be considered fair, and one where it would be considered unfair. You do not need to mathematically prove your claims, but your justification should be convincing and sound.

Question 3. [10 MARKS]

Part (a) [5 MARKS]

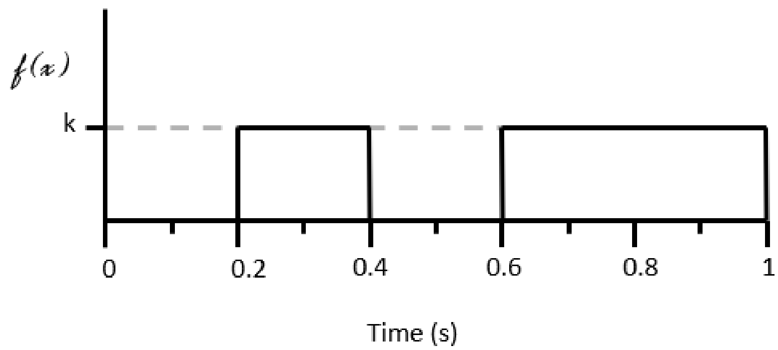
What is the expected response time of an M/M/10 system with $\lambda = 15$ and $\mu = 1.75$. Hint, this is an "open book exam". Use resources available to you. Do not do this by hand.

Part (b) [5 MARKS]

If λ became large, say 1,000,000, and $\mu = 1$, how many servers would you recommend be allocated to the system if you want the probability of a job waiting in the queue before being served to be very low (close to 0)? Assume cost is a factor, justify your answer.

Question 4. [10 MARKS]

Consider an M/G/1 system where jobs arrive with rate 1/s. Service times are distributed following the probability density function (PDF) below.



What is the expected response time?

Question 5. [10 MARKS]

Part (a) [2 MARKS]

What is a Jackson Network?

Part (b) [8 MARKS]

Describe three separate ways one could analyse a Jackson network. Give an overview of each method. What are the advantages/disadvantages each has over the others?